[N750] A Numerical Technique for the Simulation of Impulsive Noise Generation and Propagation

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ABSTRACT

Computational studies are carried out to offer validation of the technique for the simulation of impulsive sound generation and propagation developed in this paper. Our scheme developed here, a high-order Dispersion Relation Based Finite Volume Method (DRBFVM), is used to
solve the nonlinear acoustic phenomena, e.g., Shock-Sound interaction problem or numerical solutions where the mean flow is discontinuous. The coefficients used in this scheme are constructed to minimize the truncation error in the wavenumber space with less grid points per wavenumber and biased stencils for discontinuities. Several numerical results compared with the exact solutions are given, which show the good performances of this method near discontinuity. A numerical study on wave dynamic procedure occurring in muzzle flows, which are made by a supersonic projectile released from the open end of a tube into ambient air condition, is described. The Euler equations, assuming axi-symmetric flows, are is solved by using DRBFVM developed in this paper for Computational Aeroacoustics(CAA). From numerical simulation in the nearfield, the complex phenomena, including blast waves, jet flows, shock waves and their interactions in the muzzle blast, are described and impulsive noise generation and propagation resulting from complex muzzle blast is demonstrated.

**KEYWORDS:** Dispersion Relation Based Finite Volume Method(DRBFVM), Shock-Sound interaction, muzzle blast, CAA(Computational Aeroacoustics), Ffowcs Williams-Hawkings formulation(HW-H)

**INTRODUCTION**

A variety of military gun propulsion systems have been investigated and developed including those that utilize advanced solid propellant configurations[1,2]. Upon ignition and burning, the solid propellant in these systems takes on a highly complex structure that includes the dynamics of propellant combustion and various multiphase flow phenomena. The numerical study of such aeroacoustic problems places stringent demands on the choice of a computational algorithm, because it requires the ability to propagate disturbances of small amplitude and short waves. The demands are particularly high when shock waves are involved, because the chosen algorithm must also resolve discontinuities in the solution with a stiff source. The extent to which a high-order-accurate shock capturing method in multiphase reacting flows can be relied upon for aeroacoustics applications that involve the interaction of shocks with other waves has not been previously quantified[3,4]. The numerical methodology in obtaining a globally high-order-accurate solution in such a case with a shock-capturing method is demonstrated through the study of a simplified model problem. The objective of current study is to develop a numerical technique for noise prediction generated through muzzle blast and to evaluate the utility of computational models in the design of large-scale propellant systems. The current simulations correctly capture both the levels and nonlinear characteristics of the discontinuous acoustic signals.
NUMERICAL METHODS

The objective of the present study is to develop a numerical technique with minimum errors of dispersion and dissipation for aeroacoustic applications. A direct application of classic DRP methodology to the finite volume formulation is difficult since DRP scheme with the central differencing approach tries to model spatial derivatives as accurately as possible. Especially, the proposed approach must guarantee highly interacting complex flow informations such as muzzle blast and propulsion.

The governing equations are written in two dimensions for the sake of simplicity. In order to achieving high spatial accuracy in a finite volume scheme, high-order interpolation formulas for each component of the vector $Q_j$ are used as follows.

$$
\tilde{Q}_{j\frac{1}{2}}^{L,op} = \sum_{k=1}^{3} w_{L,k} \tilde{Q}_{j\frac{1}{2}}^{L,k}
$$

$$
\tilde{Q}_{j\frac{1}{2}}^{R,op} = \sum_{k=1}^{3} w_{R,k} \tilde{Q}_{j\frac{1}{2}}^{R,k}
$$

(1)

where $w_{L(R),k}$ $(k = 1,2,3)$ are the weighting coefficients and variables at cell interface, $Q_{j\frac{1}{2}}^{R,op}$ is a linear combination of function $Q_{j\frac{1}{2}}^{R,k}$ to be more than fourth-order accurate. Each coefficients are determined on condition that the dispersion relation is maintained in all space and for all time.

Taking the Fourier transform of both sides of Eq. (1) yields

$$
\tilde{Q}(\alpha) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Q(x) e^{-i\alpha x} d\alpha
$$

$$
Q(x) = \int_{-\infty}^{\infty} \tilde{Q}(\alpha) e^{i\alpha x} d\alpha
$$

(2)

(3)

$$
\tilde{Q}(\alpha \Delta \xi) \approx \frac{1}{\Delta \xi} \left[ \sum_{k=1}^{3} w_k e^{i(\alpha \Delta \xi) \Delta k} \right], \Delta_k \in \left\{ \frac{5}{2}, -\frac{3}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots \right\}
$$

(4)

It is desirable to ensure that the Fourier transform of the finite difference is a good approximation to the partial derivative over the range of wave numbers of interest. Therefore such purpose can be achieved by minimizing the integrated error $E$, defined by the following:

$$
E = \int_{0}^{\sigma} \left\| \frac{\partial}{\partial \tilde{x}} \Delta \tilde{x} - \alpha \Delta \tilde{x} \right\|^2 d(\alpha \Delta \tilde{x}) + \int_{0}^{\sigma} \left| \int_{0}^{\tilde{x}} + \lambda \int_{0}^{\tilde{x}} + Sgn(c) \exp \left[ -\ln 2 \left( \frac{\alpha \Delta \tilde{x} - \pi}{\sigma} \right)^2 \right] \right| d(\alpha \Delta \tilde{x})
$$

(5)
NUMERICAL SIMULATIONS

Two problems are chosen for the validation process. The first example is a model problem for the shock/turbulence interaction and the second example is a model problem for nonlinear blast wave/acoustic interaction and high order accuracy of numerical technique, which is selected from Third Computational Aeroacoustics (CAA) Workshop on Benchmark Problems.

Shock and Sine Wave Interactions

The governing equations are the one-dimensional Euler equations with the initial condition

\[
\begin{align*}
(\rho, u, p)_{L} &= (3.85714, 2.62936, 10.33333) \quad x < -4 \\
(\rho, u, p)_{R} &= (1 + 0.2 \sin(5x), 0.0, 1.0) \quad x \geq -4
\end{align*}
\]

where \( \rho, u, \) and \( p \) are density, velocity, and pressure, respectively. This example contains both shocks and fine structures in smooth regions, a simple model for shock-turbulence interactions. The computational domain is \([-5,5]\] and a mesh of size \( \Delta x = 1/40 \) is used.

Shock-Sound Interaction Problem

The high-order accurate scheme as well as a good performance of shock chaptering is especially important to simulate nonlinear or discontinuous wave propagations. To test these requirements, numerical shock-sound interaction problem which is suggested in third CAA Benchmark Problem is often carried out. To simulate the shock-sound interactions, the problem is simplified as a sound wave passing through a shock in a quasi-1D supersonic nozzle. In this problem, the quasi-1D Euler equations are solved:
\[
\frac{\partial}{\partial t} (\rho A) + \frac{\partial}{\partial x} (\rho u A) = 0 \\
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) + \frac{\partial p}{\partial x} = 0 \\
\frac{\partial}{\partial t} (\rho A) + \frac{\partial}{\partial x} (p A) + (\gamma - 1) p \frac{\partial}{\partial x} (u A) = 0
\]  
(7)

The detailed contents such as the area of the nozzle A and inlet/outlet boundary conditions can be referred in ref[7]. After steady state solution is achieved, an acoustic disturbance is introduced at the inlet boundary region. Figure 2 illustrates the perturbation over the period of the perturbation. The acoustic wave propagates to the shock and a reflected/transmitted waves are formed in this region. It is remarkable that a large magnitude of variables is generated near the shock position due to the interaction between the acoustic wave and the shock wave.

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\[\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} + \frac{S}{r} = 0,\]  
(8)

where \( U, F, G \) and \( S \) denote the state variables, fluxes, and source, respectively. In these equations, flux vector splitting according to Steger and Warming is utilized with the use of numerical approach suggested in this paper. Numerical solutions were marched in time by using the Runge-Kutta method of second-order accuracy. Figure 3 shows the density contour at the moving projectile speed of \( M = 1.2 \). In muzzle blast problems, acoustic field and large-scale flow field must be solved simultaneously to resolve interaction between shock and acoustic waves in near field. This high order numerical approach can guarantee the propagation of acoustic variables in the far-field as well as implement large scale interaction.

![Figure 2: The shock-acoustic interaction by an acoustic disturbance from inlet boundary region.](image)
between shocks and acoustic waves. In this figure, it is obvious that the numerical technique represents muzzle flow phenomena such as primary blast wave, shock structure and jet core flow in the near field. There is good quantitative agreement of numerical and experimental results. As mentioned above, this methodology ensures the small scale acoustic wave and flow/acoustic interaction. Therefore, it is reliable that the results in the far field will represent impulsive noise.

The source information calculated in this process can be used to predict the sound pressure levels in the far-field with combining improved nonlinear FW-H formulation. This formulation is expected to be improved and applied to this muzzle blast problem in next study.

**CONCLUSIONS**

Dispersion Relation Based FVM(DRBKVFM) was proposed and developed as a powerful numerical tool to simulate the shock-sound interaction or the wave propagation near discontinuities. Through the comparison of the computational results with the analytic solution, the accuracy and the performance of the scheme was investigated. One advantage of this DRBFVM is that there is no need to consider the artificial damping to obtain the best results for a particular problem at hand. Another advantage is that it can be used in both the continuous and the discontinuous regions and it is possible to obtain oscillation-free numerical solutions. Furthermore, this numerical technique can guarantee accurate numerical implementation of source region such as nonlinear flow/acoustic interaction in the near field.

*Figure 3: The density distribution of the muzzle blast at the projectile speed of $M=1.2$*
as well as long distance acoustic wave propagation characteristics without dispersion and dissipation in the far field.

REFERENCES